

# VISCOUS FLOWS THROUGH SCREENS NORMAL TO THE UNIFORM STREAM

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Viscous flows through the screen normal to the uniform flow are numerically simulated and measurements of the mean velocity are made. Reynolds averaged Navier-Stokes equations are solved with a standard  $k-\epsilon$  model. The existence of screen is regarded as extra sources in the momentum equations. The amount of extra source is related to the resistance coefficient and the refraction coefficient of the screen. Elliptic type of equations are solved for 2 dimensional flow, and the partially parabolic equations for 3 dimensional flows. Wakes behind 3 dimensional mesh-screens of various configurations have been measured in the cavitation tunnel. The present numerical method is verified to reasonably simulate the viscous wake of the screen, for which the inviscid theory is quite limited. Considerable attenuations of the viscous shear layer in the wake of the screen are experimentally observed and numerically simulated. A detached separation-bubble from the two-dimensional screen is simulated as the resistance coefficient is increased to a certain level.

**Key Words :** Screen, Elliptic Equation, Partially Parabolic Equation,  $k-\epsilon$  Model, Resistance Coefficient, Refraction Coefficient, Separation Bubble

## NOMENCLATURE

$A_i, B$  : Coefficients in discretized equation  
 $C_i$  : Turbulence model constants  
 $dA$  : Area of screen element  
 $G$  : Generation of  $k$   
 $H$  : Total head  
 $H_0$  : Half width of duct  
 $H_s$  : Half height of screen  
 $K$  : Resistance coefficient of screen  
 $k$  : Turbulent kinetic energy  
 $p$  : Static pressure  
 $q$  : Velocity vector  
 $Re$  : Reynolds number ( $W_0 H_s / \nu$ )  
 $S_\phi$  : Source terms of subscripted entity  
 $U, V, W$  : Velocity components in  $x, y, z$ -directions  
 $x, y, z$  : Cartesian coordinates  
 $\alpha$  : Refraction coefficient of screen  
 $\epsilon$  : Dissipation rate of  $k$   
 $\Gamma$  : Diffusion coefficient  
 $\mu$  : Viscosity  
 $\mu_{eff}$  : Effective viscosity  
 $\mu_t$  : Turbulent viscosity  
 $\omega$  : Vorticity vector  
 $\phi$  : Unknown variables  
 $\rho$  : Density  
 $\sigma_k, \sigma_\epsilon$  : Schmidt number of subscripted entity  
 $\theta$  : Incoming angle of flow to screen

## Subscripts

$d$  : Downstream surface of screen  
 $o$  or  $-o$  : Far upstream of screen  
 $+o$  : Far downstream of screen  
 $P, E, W, N, S$  : Grid points  
 $s$  : Screen  
 $u$  : Upstream surface of screen

## 1. INTRODUCTION

Control of the velocity distribution of a fluid flow is fundamental and important problem in engineering fluid mechanics. The composite flow must be reproduced for tested results to be meaningful. One of the problems is to generate required mean velocity and turbulence distribution in either a wind tunnel or a cavitation tunnel. The performance and cavitation tests of the propeller are made in a properly scaled non-uniform wake of the ship.

A screen may be used to generate a flow field. It may be thought of as any distributed resistance which change flow direction and reduce pressure. Common examples of screen are an array of parallel rods, honeycombs, perforated plates and wire-mesh screens. Some reviews on the topic have been made by Laws and Levesey(1978). Since Taylor and Batchelor(1949), Owen and Zienkiewicz(1957), and Elder(1959) have formulated weakly-sheared two dimensional flows past wire grids. McCarthy(1964) extended the problem to three-dimensional field without placing restrictions on the magnitude of variation of resistance across the screen. A higher-order solution of the partial linearization of the equations of motion was found, which gives a non-linear relation between

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downstream velocity distribution and grid resistance coefficient distribution. Koo and James(1973) have suggested a mathematical model for two dimensional flow past partially filled screen in the channel. Turner(1969), Livesey and Laws(1973a, 1974b) have also made further developments of past researches.

Estimation of longitudinal variation of the mean velocity and the turbulent intensity are important in the ship-wake generation problem. Furthermore valuable information about the effective wake component can be estimated by considering the interaction between the screen and the propeller. But inviscid theories are not able to describe such viscous phenomena. Numerical simulation of turbulent flow past a screen is tried, and results are compared with those of inviscid calculation and experimental data obtained in the cavitation tunnel in the present study.

## 2. INVISCID THEORY AND SCREEN

For steady incompressible flow past a screen of non-uniform property, it is assumed that viscosity effects are negligible except over a screen. A screen is installed at  $z=0$  plane in Fig. 1. Static pressure and lateral velocity components at far upstream and far downstream of the screen are taken to be zero. A screen is considered to be a surface of hydrodynamic discontinuity. On passing through the grid, the flow is locally refracted and static pressures are reduced. The discontinuity of lateral velocity through the screen is described by the refraction coefficient.

$$\alpha = \left( \frac{U_d^2 + V_d^2}{U_u^2 + V_u^2} \right)^{\frac{1}{2}} \quad (1)$$

The resistance coefficient is defined by the non-dimensionalized pressure drop through the screen.

$$K = \frac{(p_u - p_d)}{(1/2)\rho W_s^2} \quad (2)$$

$W_s$  denotes the velocity normal to the screen in Fig. 2. The problem is to find the relationship between flows far upstream and downstream of the screen and resistance and refraction characteristics of the screen. McCarthy's theory for 3 dimensional flows is introduced here.

The governing equation for inviscid flow is written as follow.

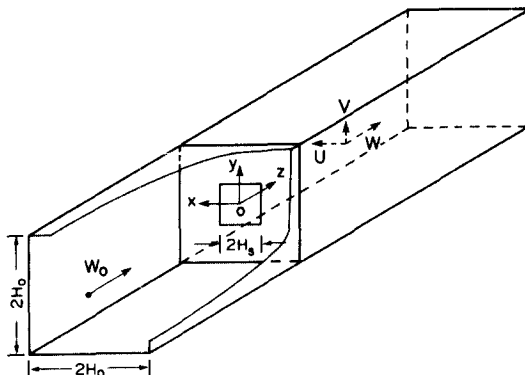


Fig. 1 Screen alignment in the duct and coordinate system

$$\nabla H = q \times \omega \quad (3)$$

Assuming that streamline deflexions are sufficiently small, vorticity is considered to be constant along streamlines. Then the vorticity components may be written in terms of the velocity components far upstream and downstream of the screen. All changes in vorticity is confined to the screen, i.e. the surface of hydrodynamic discontinuity. Considering the continuity  $W_u = W_d = W_s$  on the screen, the following form of equation is obtained.

$$K_d W + \frac{1}{2} W_s dK + d(W_{+o} - W_{-o}) = 0 \quad (4)$$

It is assumed that the disturbed flow from the basic flow at  $z = -o$  and  $z = +o$  are potential flows. Then a relation between  $W_{+o}$ ,  $W_{-o}$  and  $W_s$  is obtained(McCarthy, 1964).

$$W_s = \frac{W_{+o} + \alpha W_{-o}}{1 + \alpha} \quad (5)$$

If  $W_{-o}$ ,  $K$  and  $\alpha$  are given,  $W_{+o}$  can be obtained. Final form of McCarthy's theory is given as follows ;

$$\frac{W_{+o}}{W_{-o}} = 1 - \frac{1 + \chi}{(1 + \chi^3)^{2/3}} \left[ \frac{1/6 + \chi^3}{\chi^2} - \gamma_0 \right] \quad (6)$$

where

$$\chi = (1 + K)^{1/2}, N = 1.02, \\ \gamma_0 = \int_A \frac{1 + \chi}{(1 + \chi^3)^{2/3}} \frac{1/6 + \chi^3}{\chi^2} dA / \int \frac{1 + \chi}{(1 + \chi^3)^{2/3}} dA$$

A method of correction for the effect of streamline deflexion is also suggested.

$$W_i(+o)^* = \frac{\Sigma W_i(0) A_i(0) / W_i(+o)}{\Sigma W_i(0) A_i(0)} W_i(+o) \quad (7)$$

## 3. GOVERNING EQUATIONS FOR VISCOUS FLOW

The continuity equation and steady state three-dimensional Reynolds' equations in Cartesian coordinate are adopted, and a standard form of  $k-\epsilon$  model is taken in the present study. When the resistance coefficient of the screen is not so large, no region of flow reversal in the main direction is expected. Therefore the problem will be one of the cases for which the partially parabolic type of equations(Pratap and Spalding, 1979) can be applied with benefits. The equations are obtained by neglecting  $\partial^2/\partial z^2$  terms among diffusion terms. Then the following form of the governing equation is obtained.

$$\frac{\partial}{\partial x}(\rho U \phi) + \frac{\partial}{\partial y}(\rho V \phi) + \frac{\partial}{\partial z}(\rho W \phi) \\ = \frac{\partial}{\partial x} \left( \Gamma_x \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_y \frac{\partial \phi}{\partial y} \right) + S_\phi \quad (8)$$

Parameters in the equation are summarized in Table 1.

The screen is immersed in the calculation domain, and its treatment is explained in the next section. The boundary of the flow domain is composed of the inlet plane far upstream, the outlet plane far downstream of the screen, and the duct wall. When a symmetric screen is installed at the center of

**Table 1**  $\phi$ ,  $\Gamma_\phi$  and  $S_\phi$  for governing equations

$\phi$	$\Gamma_\phi$	$S_\phi$
1	0	0
$U$	$\mu_{eff}$	$S_u = \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial W}{\partial x} \right)$
$V$	$\mu_{eff}$	$S_v = \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial W}{\partial y} \right)$
$W$	$\mu_{eff}$	$S_w = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial U}{\partial z} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial V}{\partial z} \right) + \frac{\partial}{\partial z} \left( \mu_e \frac{\partial W}{\partial z} \right)$
$k$	$\frac{\mu_{eff}}{\sigma_k}$	$G - C_{D\rho\epsilon}$
$\epsilon$	$\frac{\mu_{eff}}{\sigma_\epsilon}$	$\frac{\epsilon}{k} (C_1 - G - C_2\rho\epsilon)$

the duct, the symmetric plane is one of its boundaries. Incoming flow is uniformly distributed. It is assumed that  $k = 0.001 W_0^2$  and  $\epsilon = C_\mu k^{3/2} / 0.005 H_0$ . Neumann condition is adopted at the exit-plane. Boundary layer development on the duct-wall is not taken into account through the study. All the quantity are assumed to be symmetric on the wall as well as on the plane of symmetry, except that velocities normal to the wall and the plane of symmetry are equal to zero.

where  $G = \mu_t \left\{ 2 \left[ \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 + \left( \frac{\partial W}{\partial z} \right)^2 \right] + \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 + \left( \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right)^2 \right\}$

$\mu_{eff} = \mu + \mu_t$ ,  $\mu_t = C_\mu \rho k^2 / \epsilon$   
 $C_\mu = 0.09$ ,  $C_D = 1.0$ ,  $C_1 = 1.44$ ,  
 $C_2 = 1.92$ ,  $\sigma_k = 1.0$ ,  $\sigma_\epsilon = 1.3$

## 4. SCREEN MODEL AND NUMERICAL METHOD

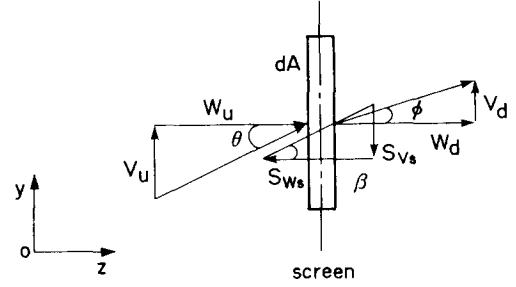
It is assumed that the existence of the screen is creating extra momentum sink in each corresponding direction. The amounts of sink are expressed by flow characteristics of the screen. The relations are obtained by integrating the momentum equations over a thin control volume including the screen element as described in Fig. 2. From the  $x$ -momentum equation, we obtain

$$-\rho W_u^2 dA + \rho W_d^2 dA = p_u dA - p_d dA - S_{ws} dA \quad (9)$$

Since  $W_u = W_d = W_s$  by the continuity, the amount of sink in the main direction is given by

$$S_{ws} = \frac{K}{2} \rho W_s^2 \quad (10)$$

By considering the conservation of momentum in lateral directions,

**Fig. 2** Velocity components on the  $y$ - $z$  section of a screen element

$$S_{us} = \rho W_u U_u - \rho W_d U_d \quad (11)$$

$$S_{vs} = \rho W_u V_u - \rho W_d V_d \quad (12)$$

$S_{ws}$ ,  $S_{us}$  and  $S_{vs}$  in the above equations are equivalent to each components of resistance of the screen per unit area. Since the direction of resultant drag is not known, it is assumed that it is parallel to the incoming flow direction  $\theta$  on the screen. A refraction coefficient  $\alpha$  is a function of  $K$ ,  $\theta$  and Reynolds number. The empirical relation of Taylor and Batchelor(1949) is adopted here.

$$\alpha = \frac{2\theta}{\sin 2\theta} \frac{1.1}{(1 + K \cos^2 \theta)^{1/2}} + 1 - \frac{2\theta}{\sin 2\theta} \quad (13)$$

Amounts of discontinuity in  $k$  and  $\epsilon$  across the screen are difficult to be estimated. Effects of guessed values of  $k$  and  $\epsilon$  on the predictions have been studied. Self-generated turbulence energy in the shear layer along the screen boundary is dominant except at the near-wake region of the screen(Kang and Jeon, 1988). Hence we assumed that  $k = 0.01 W_s^2$  and  $\epsilon = C_\mu k^{3/2} / l$ . The length scale  $l$  is given as  $0.01 H_s$ .

The finite difference equations are obtained by integrating equations over individual control volumes formed by the staggered grids system. According to the finite volume approach with the hybrid convective differencing scheme, the following general form of discretized equation is obtained(Johnson, 1984).

$$A_P \phi_P = A_E \phi_E + A_W \phi_W + A_N \phi_N + A_S \phi_S + C_u \phi_u + B \quad (14)$$

Since  $\phi_P$  is not directly dependent upon downstream nodal values  $\phi_d$ , a parabolic marching procedure along the  $z$ -direction can be employed. But this procedure is not truly parabolic in the  $z$ -direction, because the longitudinal pressure gradient appears in the source term of the momentum equations. Downstream effects can reach upstream through the pressure field of elliptic characteristic. Once the pressures are assumed or estimated,  $U$ ,  $V$ ,  $W$ ,  $k$ ,  $\epsilon$  can be calculated using upstream values. At each cross-sectional plane, TDMA and the line-by-line method are adopted to solve the algebraic equations (Patakar, 1980).

SIMPLE algorithm is used to get the finite difference pressure correction equation, which has fully elliptic nature. Pressure corrections at a given section are made with upstream and downstream pressures fixed during a marching procedure. But the bulk pressure correction and the upstream pressure correction scheme are used to speed up the convergence rate, when new pressures are calculated at the section (Johnson, 1984). Several sweeps in the  $z$ -direction are required obtain the converged solution.

### 5. EXPERIMENTS, CALCULATIONS AND DISCUSSIONS

Mean velocity distributions through several wire-mesh screens have been measured by rotating a pitot tube rake in the cavitation tunnel, which has the square section of  $0.85\text{m} \times 0.85\text{m}$ . Measurements were carried out at the uniform velocity of  $4.5\text{m/s}$ . A circular disk of screen and two square plate-screens composed of symmetrically and asymmetrically aligned rectangular strips of various solidities are designed. Three values of solidity are obtained by folding a base wire-mesh. The specification of the base wire-mesh and configurations of three screens adopted in this study are sketched in Fig. 3. Measured resistance coefficients of one-, two-, and three-folded mesh sheet were  $0.53$ ,  $0.97$ , and  $1.44$  respectively (Kang and Jeon, 1988).

One quarter of the duct section is divided into  $22 \times 22$  non-uniform grids by considering the symmetry of the flow field, if applicable. The flow domain is extended from the upstream of  $4H_s$  to the downstream of  $12H_s$ . 42 grids are allocated in the longitudinal direction. The turbulent kinetic energy  $k$  and the length scale  $l$  at the downstream surface of the screen are assumed to be  $0.1W_s^2$  and  $0.01H_s$ .

Measured and calculated mean velocity distributions through the circular uniform screen of a base mesh are compared in Fig. 4. The velocity is measured from the screen to one-duct-width downstream, where the propeller performance test is usually executed. Slight scatterings in measured velocity profiles appear near the screen. Velocities gradually

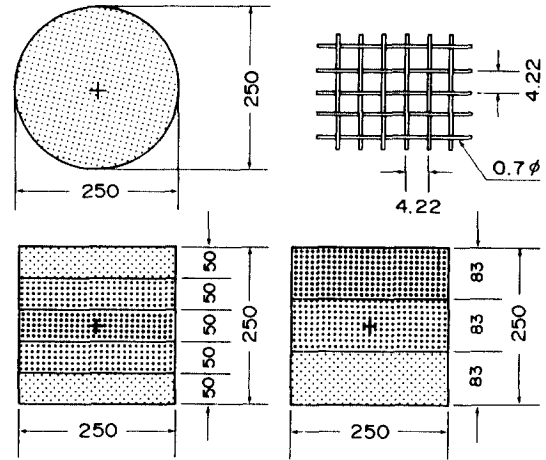


Fig. 3 Configurations of wire-mesh screens

change from the center to the wall of the duct except in the thin shear layer along stream lines passing the edge of the screen. The present viscous flow calculation nicely simulates the whole flow through the screen. Predictions according to McCarthy's inviscid theory are also in good agreement with the measurements. But the inviscid theory cannot describe the formation of the shear layer along downstream. Measured values of the velocity past the two-folded circular screen show more scatterings near the screen and at the external region. But the velocity profile at the downstream is still well-simulated. As the solidity is increased, estimated

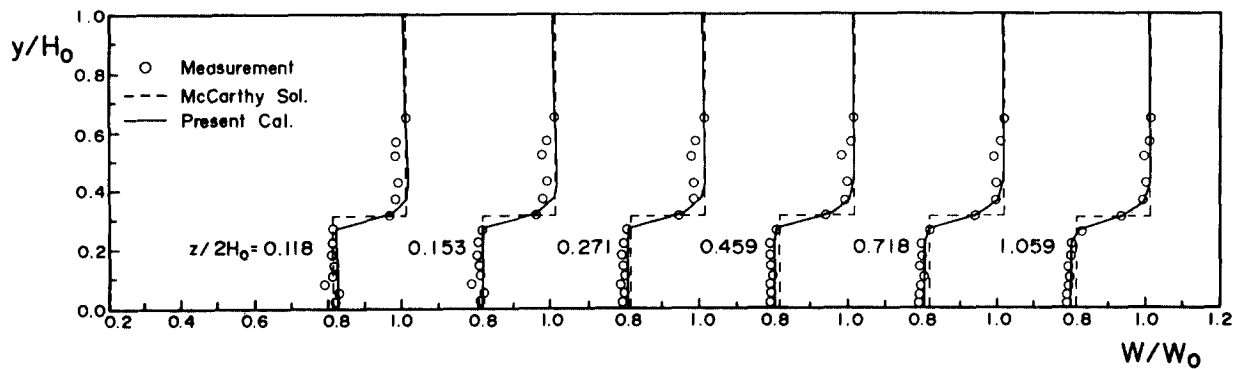


Fig. 4 Distributions of the longitudinal mean velocity through the one fold circular screen

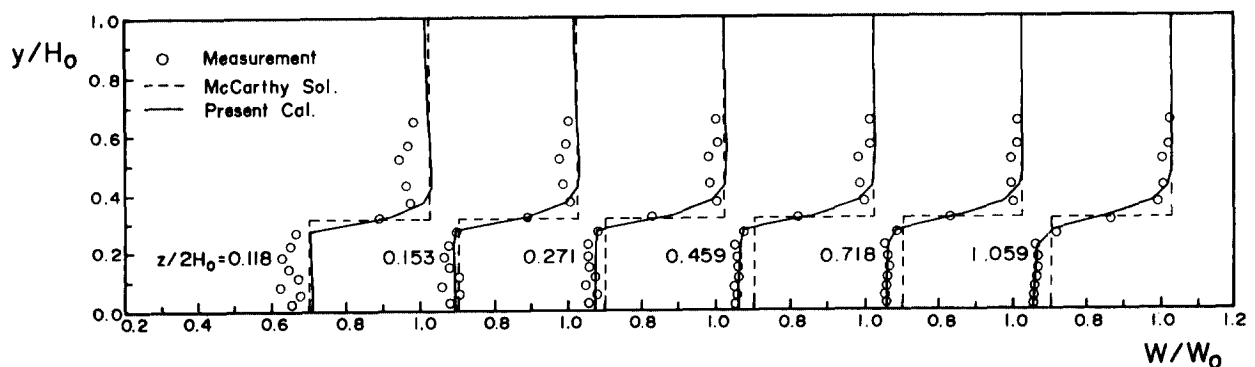


Fig. 5 Distributions of the longitudinal mean velocity through the two folded circular screen

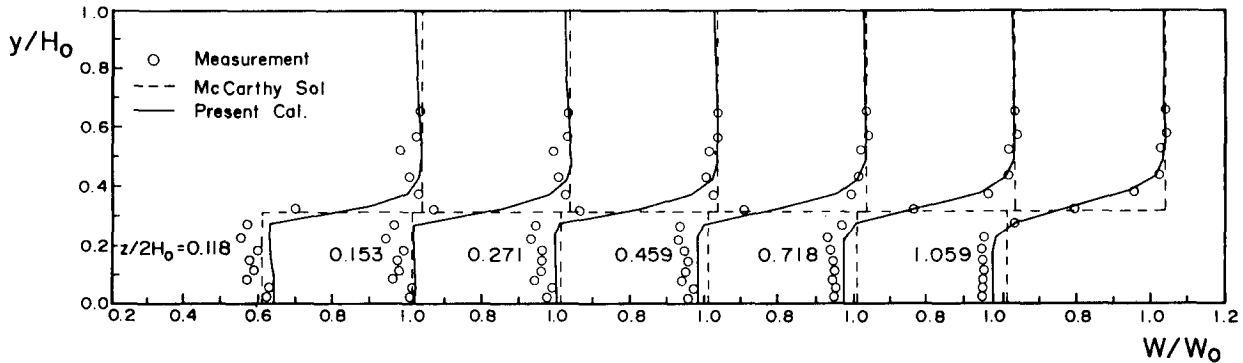


Fig. 6 Distributions of the longitudinal mean velocity through the three folded circular screen

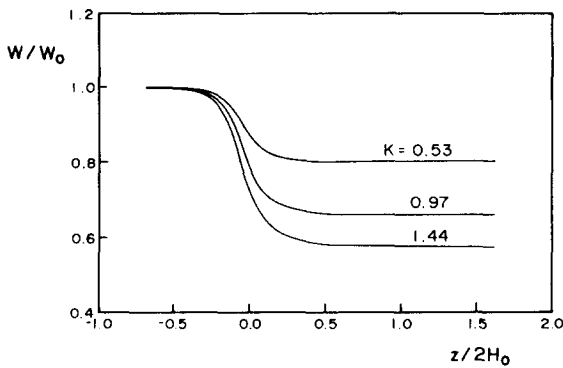


Fig. 7 Variations of the longitudinal velocity through circular screen along the center of the duct

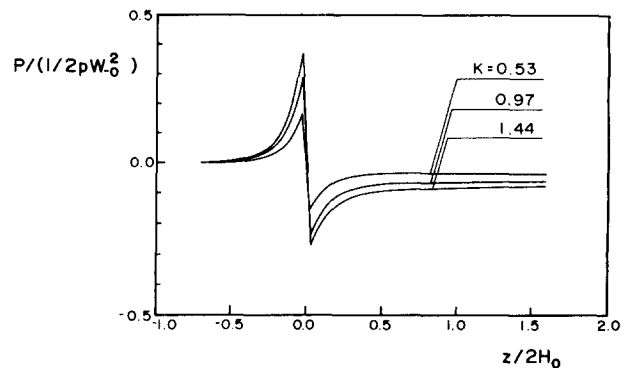


Fig. 8 Variations of the static pressure through circular screens along the center of the duct

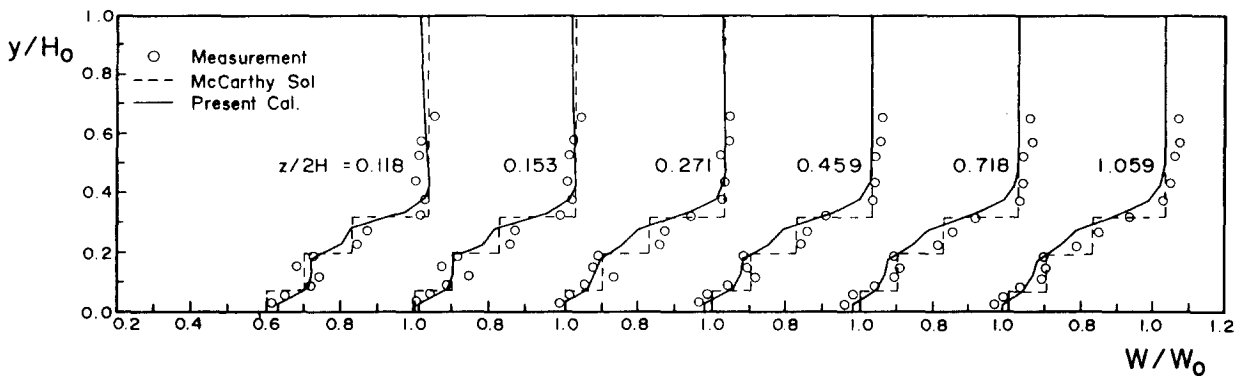


Fig. 9 Distributions of the longitudinal velocity through the symmetric square screen

values by the inviscid theory are considerably deviated from the measured values. In case of the three-folded meshes, calculated results show some deviations at the inner wake region. Such over-estimations may be originated from the uncertainty of the resistance coefficient, since the physical condition of the three folded mesh might be changed and consistent deviations at each cross section appear in Fig. 6. Figure 7 and Fig.8 display variations of the calculated longitudinal velocity and the static pressure along the center line of the duct. The velocity is gradually reduced and approaches to the constant value at the one-duct-width downstream. The discontinuity in static pressures appear as expected in Fig. 8. Bernoulli's equation is almost satisfied at the upstream region of the screen. At downstream of the screen pressures are recovered with about 0.5% relative loss in total head,

since the strain of mean flow is negligible at the central region of the wake. The increase in the pressure upstream of the screen can be simulated by the characteristic of the partially parabolic method, which allows the down stream effects on the flow upstream by solving pressures elliptically. The present method successfully describe the attenuation of the generated wake field, which is useful on planning the propeller performance test in the tunnel.

Flows through the square screen of five strips are presented in Fig. 9. Though the accordance of the simulated results with the measured ones are not as good as the above cases of uniform solidity, the simulation is still generally rational. Velocity distributions through the square screen composed of three asymmetric strips are shown in Fig. 10. The trend is nearly the same as the symmetric one. Considerable scatter-

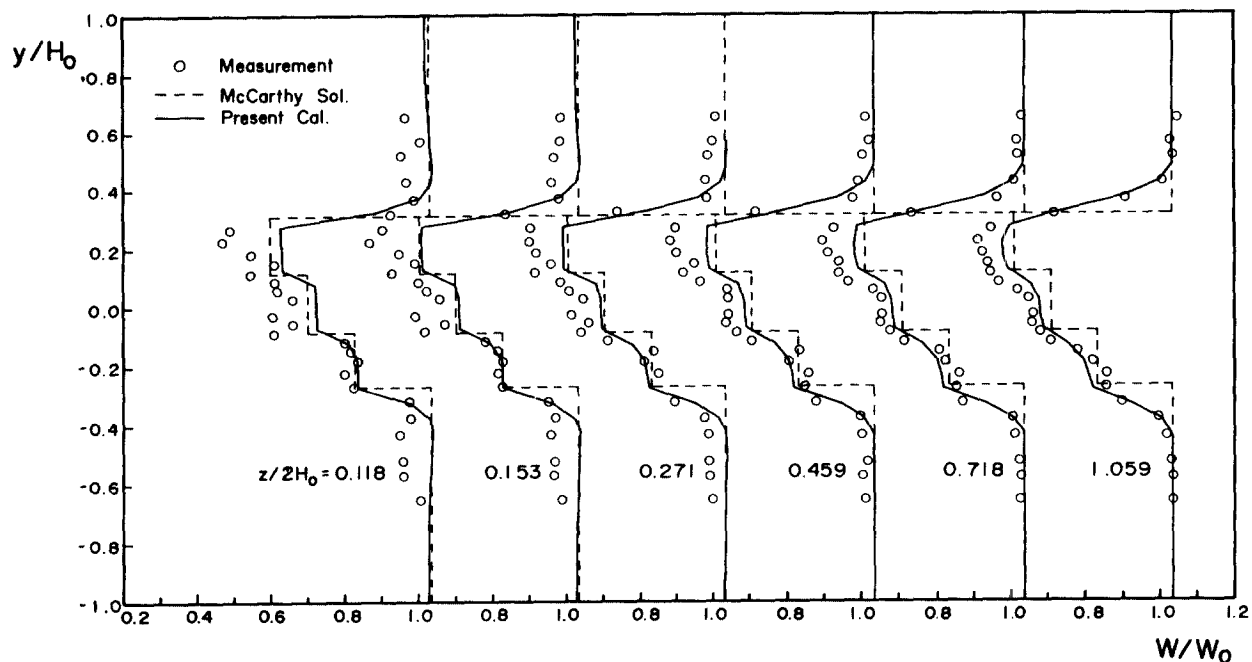


Fig. 10 Distributions of the longitudinal velocity through the asymmetric square screen

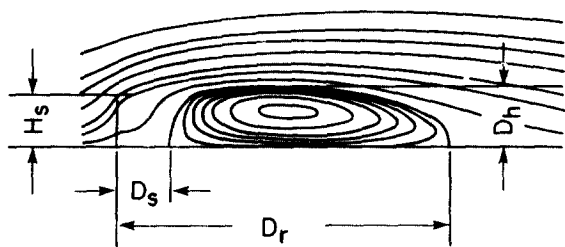


Fig. 11 Configuration of a recirculating bubble

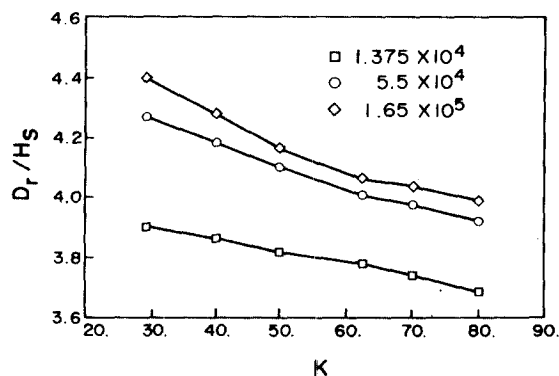


Fig. 13 Variations of the reattachment point of the bubble

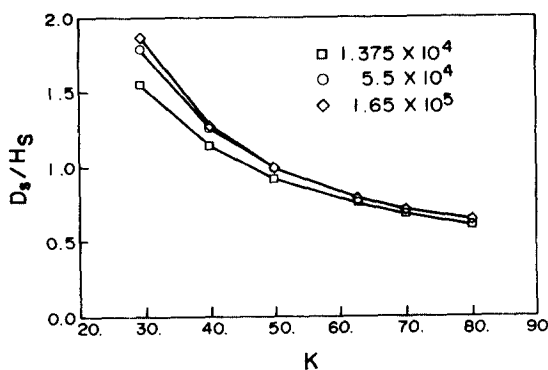


Fig. 12 Variations of the separation point of the bubble

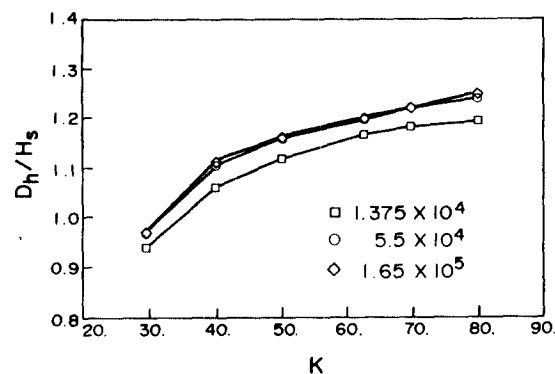


Fig. 14 Variations of the height of the recirculating bubble

ing in the measured data seems to be originated from high turbulence generated from discontinuous solidity of strips. As pointed out above, uncertainty in resistance coefficient is also one of factors which make deviations in the prediction.

Flows through the two dimensional highly dense screen

installed at the center of the duct are simulated in the present study. Recirculating flow behind the vertical fence are well known. Similar separated flow may be expected past the screen of high solidity. The most important factors governing flow are : the resistance coefficient  $K$ , the Reynolds number

$Re = W_0 H_s / \nu$ , and the height of the screen  $H_s / H_0$ . A recirculating bubble detached from the screen shown in Fig. 11 was reported in flow through the perforated screen (Castro, 1971) and the grid screen (Honji, 1973a, 1973b). Various flows have been simulated with several values of those factors (Kang and Jeon, 1988). The bubble is formed according to the force balance between the inertia force through the screen and the adverse pressure gradient downstream of the screen. Such a flow structure is an interesting one for practical engineering application. Variations of the position and the size of the bubble with  $K$  and Reynolds numbers are calculated and presented in Fig. 12~Fig. 14. The height is taken to be a fixed value of 1/10 in the present study. As the resistance coefficient  $K$  is reduced and Reynolds number is increased, the force of inertia relative to the pressure force is increased. Therefore the bubbles move downward from the screen, and disappears in certain conditions.

## 6. CONCLUSIONS

(1) The partially parabolic solution of the Reynolds equations with a  $k-\epsilon$  turbulence model are numerically obtained. The present numerical method has been verified to reasonably simulated the viscous wake and shear layer through the wire-mesh screen, for which the inviscid theory is quite limited.

(2) Considerable attenuation of the velocity profile in the wake of the screen are experimentally observed and numerically simulated. The velocity is gradually reduced and approaches to constant value at one-duct-width downstream. Bernoulli's equation is nearly satisfied at the upstream region of the screen. At downstream of the screen pressures are recovered with minor losses in the total head.

(3) A detached separation-bubble from the screen is numerically simulated as the resistance coefficient is increased to a certain level. Such results are qualitatively in agreement with experimental data available.

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